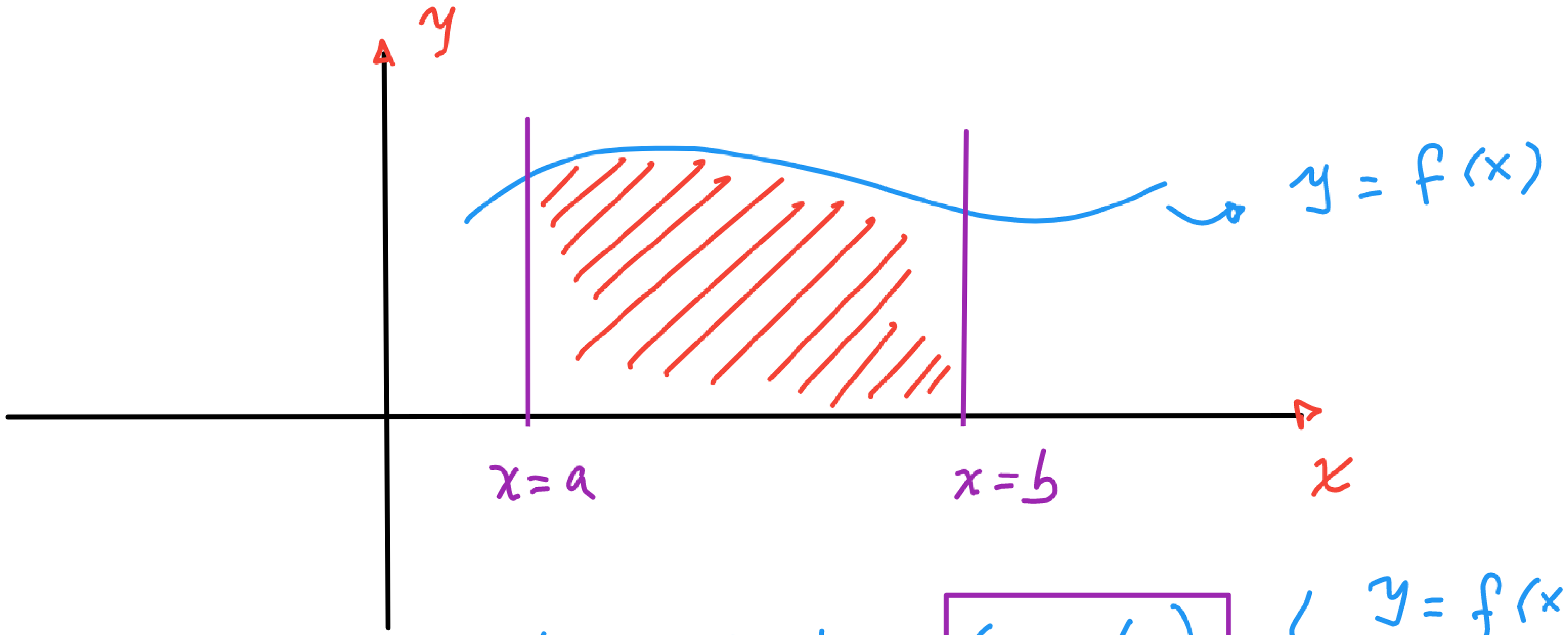
miércoles 22 de junio de 2022

Cálculo Integral



$y = f(x) \geq 0$, $\forall\ x \in [a, b]$, $\boxed{(a < b)}$

$$A = \int_a^b f(x)\,dx$$

$y = f(x)$ se expresa en forma paramétrica como

$$x = x(t) \qquad t \in [\xi, \eta]$$
$$y = y(t)$$

donde $\boxed{\begin{matrix} a = x(\xi) \\ b = x(\eta) \end{matrix}}$ (en algunas ocasiones $\xi > \eta$)

Luego:

$$A = \int_a^b f(x)\,dx$$

"cambio de variable"

$x = x(t) \Longrightarrow dx = \underbrace{\frac{d}{dt}(x(t))}_{x'(t)} \cdot dt$

$y = y(t)$

$$A = \int_a^b y \cdot dx$$

$$\boxed{A = \int_{\xi}^{\eta} y(t) \cdot x'(t) \cdot dt}$$

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$$\underbrace{\begin{array}{l} x = a\cos t \\ y = b \operatorname{sen} t \end{array}}_{t \in [0, 2\pi]}$$

$0 \leq x \leq a$

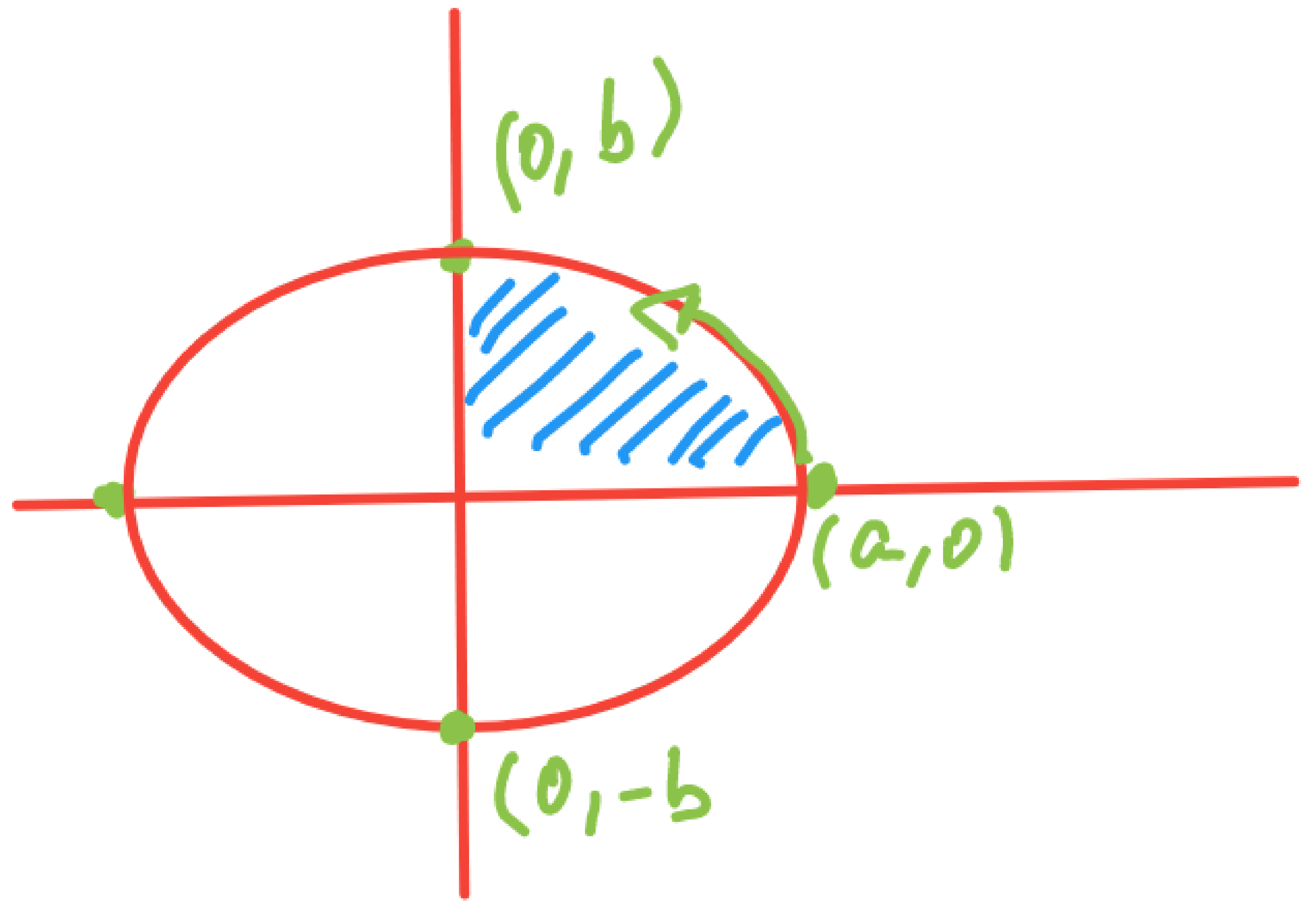
$\Downarrow$

Si $x = 0 \Rightarrow 0 = a\cos(t_1)$

$\cos(t_1) = 0 \Rightarrow \boxed{t_1 = \pi/2}$

Si $x = a \Rightarrow a = a\cos(t_2)$

$\Rightarrow 1 = \cos(t_2) \Rightarrow \boxed{t_2 = 0}$



$$A = 4\int_0^a y(x)\,dx \overset{\text{cambio variable}}{=} \boxed{4\int_{\pi/2}^{0} (b \operatorname{sen} t)(-a \operatorname{sen} t)\,dt}$$

$$A = 4ab\int_0^{\pi/2} \operatorname{sen}^2 t \, dt$$

$$A = \frac{4ab}{2}\int_0^{\pi/2} (1 - \cos(2t))\, dt$$

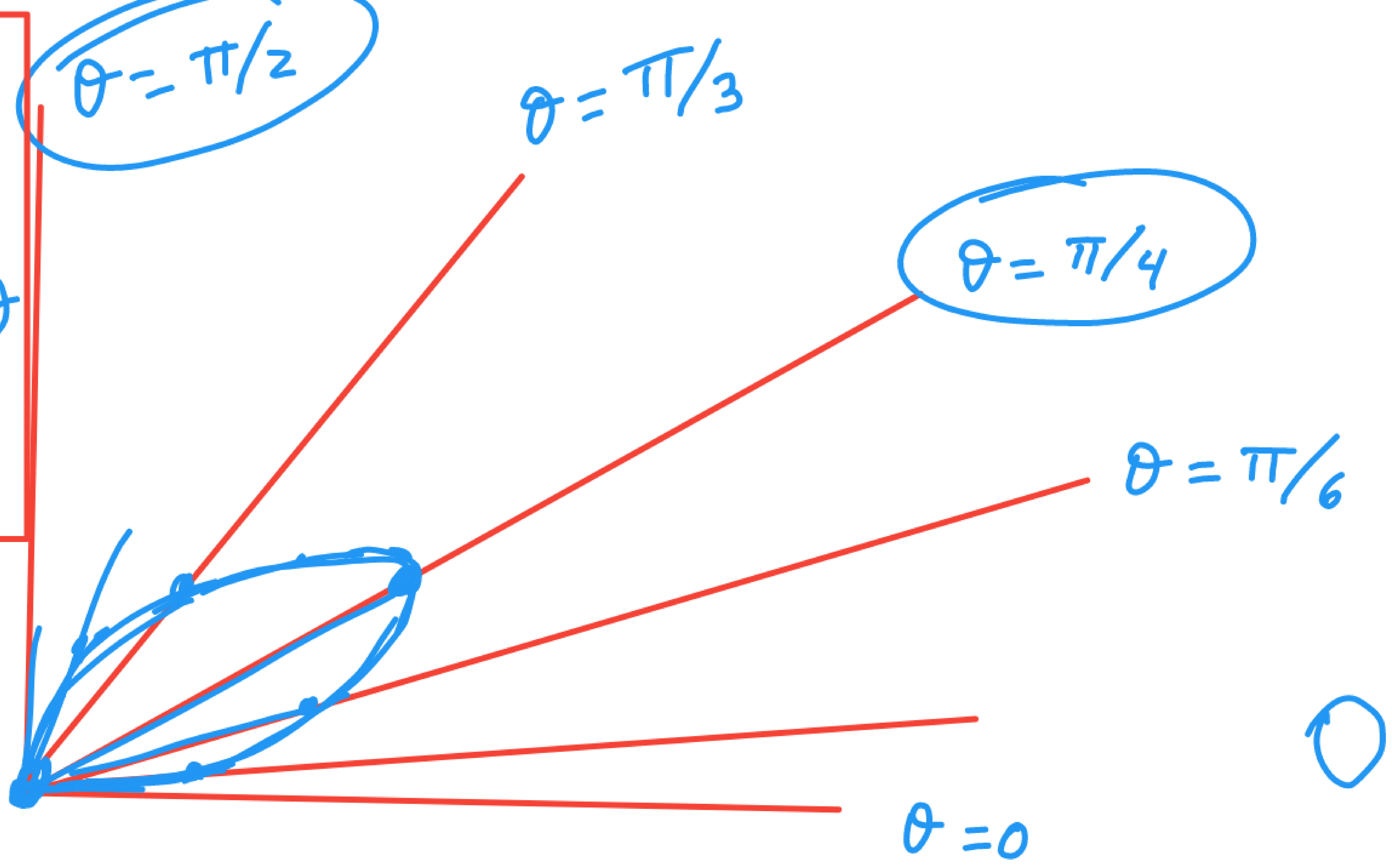
$$A = 2ab\left[t - \frac{\operatorname{sen}(2t)}{2}\right]_0^{\pi/2}$$

$$A = 2ab\left[\left(\frac{\pi}{2} - 0\right) - (0 - 0)\right]$$

$$\boxed{A = ab\pi \ u^2}$$

Borrador

$$\boxed{A = 4\left(\frac{1}{2}\right)\int_{\boxed{0}}^{\pi/2} \left(a\,\mathrm{sen}(2\theta)\right)^2 d\theta}$$



$$0 \leq \theta \leq \frac{\pi}{2}$$

$$\rho = a\,\mathrm{sen}(2\theta)$$

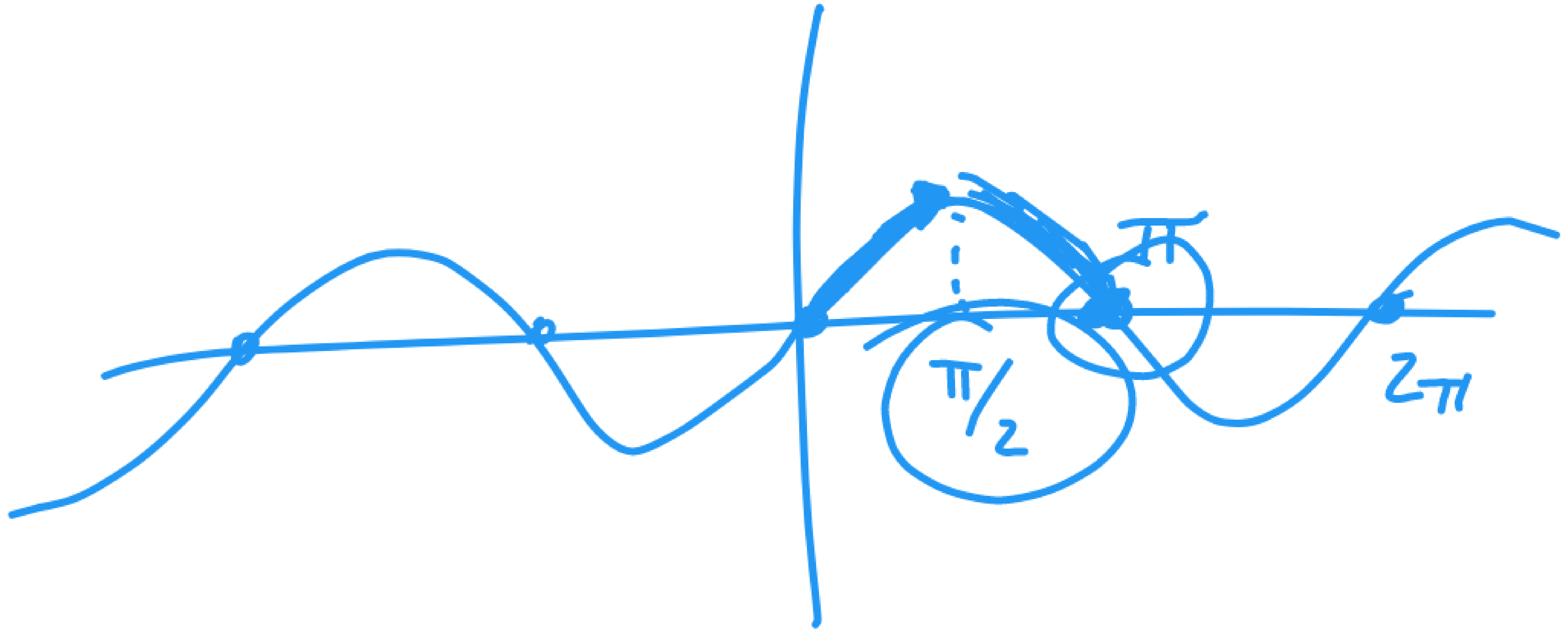
θ	ρ
0	0
$\pi/6$	.
$\pi/4$	
$\pi/2$	0

$2\theta = \pi/2$

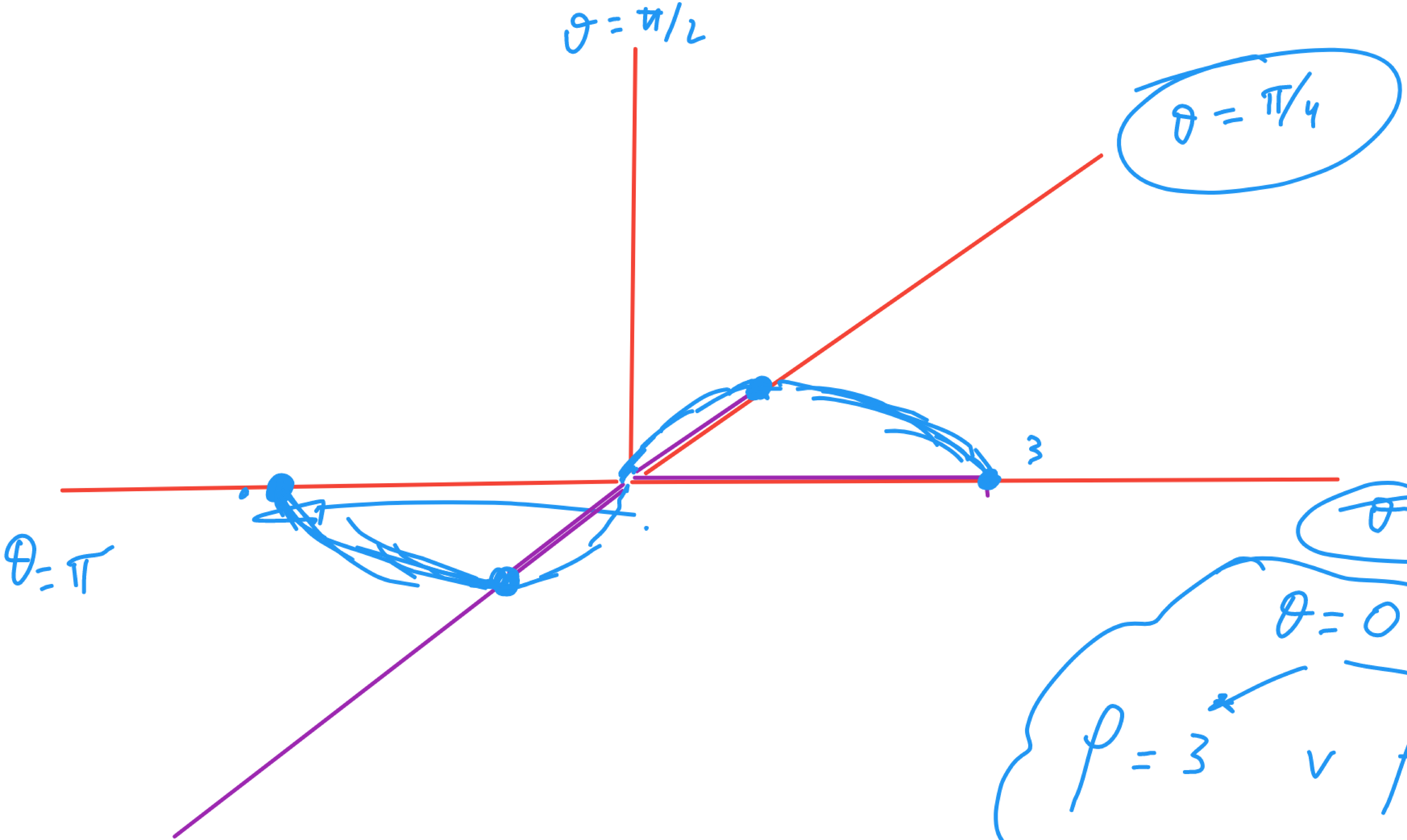
$\theta = \pi/4$

$2\theta = \pi$

$\theta = \pi/2$

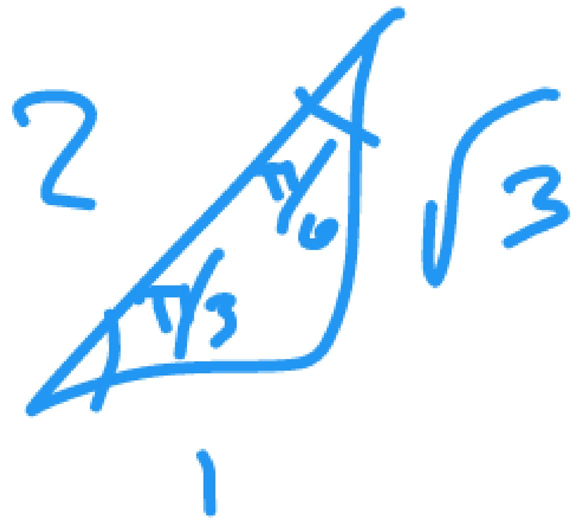


Borrador



$\theta = 0$

$\rho = 3 \quad \vee \quad \rho = -3$



$\theta = \pi/6$

$\rho = \frac{3}{\sqrt{2}}$

$\rho = -\frac{3}{\sqrt{2}}$

$\rho^2 = 9\cos 2\theta$

$\rho^2 = \frac{9}{2} \Rightarrow$

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$$A = 4\int_0^{\pi/4} \frac{\rho^2}{2}\, d\theta = 18\int_0^{\pi/4} \cos(2\theta)\, d\theta$$

$$= \frac{18}{2} \operatorname{Sen}(2\theta)\Big|_0^{\pi/4}$$

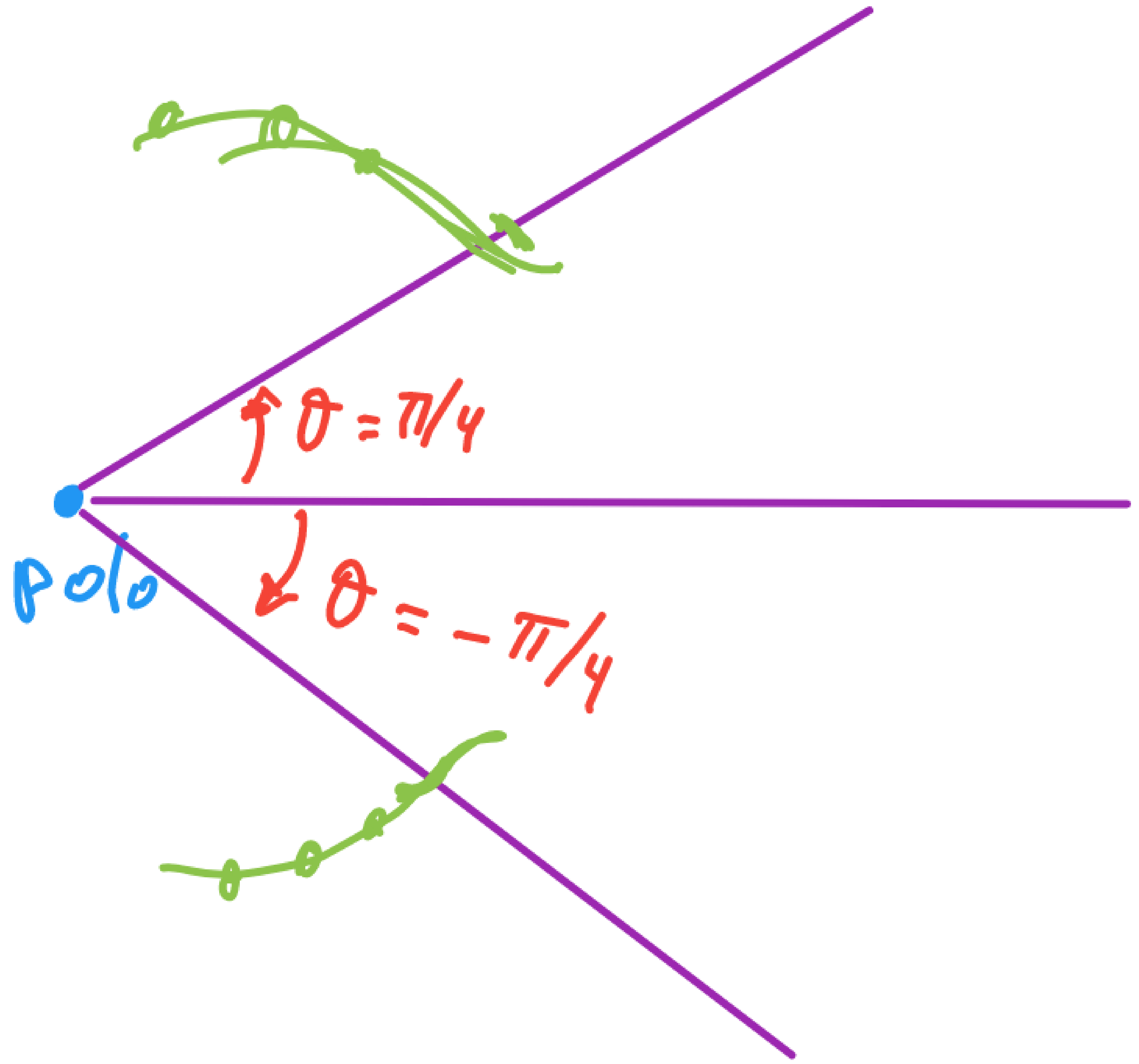
$$= 9\left[\operatorname{Sen}\left(\frac{\pi}{2}\right) - 0\right]$$

$$= 9\ u^2$$

Simetría con respecto a eje X

$\theta \to -\theta$

Si no hay cambio se observa simetría con respecto al eje $\theta = 0°$ (eje X)



Longitud de arco de una curva en coordenadas cartesianas

$y = f(x)$ continua $\forall\ x \in [a, b]$

$$L = \lim_{n \to \infty} \left(\sum_{i=1}^{n} d(P_{i-1}, P_i) \right) \Rightarrow \text{Longitud de la curva desde } A \text{ a } B$$

Ahora bien:

$$d(P_{i-1}, P_i) = \sqrt{(\underbrace{x_i - x_{i-1}}_{})^2 + (y_i - y_{i-1})^2}$$

$$= \underbrace{|x_i - x_{i-1}|}_{\Delta x_i} \sqrt{1 + \left(\frac{y_i - y_{i-1}}{x_i - x_{i-1}}\right)^2}$$

$$= \sqrt{1 + \left(\boxed{\frac{y_i - y_{i-1}}{x_i - x_{i-1}}}\right)^2} \cdot \Delta x_i \quad \ldots (*)$$

Como f es continua $\forall\, x \in [a,b] \Rightarrow$ es continua en $[x_{i-1}, x_i]$

Si además f es derivable en el abierto $]x_{i-1}, x_i[$ entonces $\Rightarrow$ existe $\xi_i \in [x_{i-1}, x_i]$ / $f'(\xi_i) = \boxed{\dfrac{y_i - y_{i-1}}{x_i - x_{i-1}}}$

En (*):

$$d(P_{i-1}, P_i) = \sqrt{1 + \left(\frac{y_i - y_{i-1}}{x_i - x_{i-1}}\right)^2} \cdot \Delta x_i$$

$$= \sqrt{1 + \left(f'(\xi_i)\right)^2} \cdot \Delta x_i$$

Luego, por definición:

$$L = \lim_{n \to \infty} \left(\sum_{i=1}^{n} \left(\boxed{\sqrt{1 + \left(f'(\xi_i)\right)^2}} \cdot \boxed{\Delta x_i} \right) \right) = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

$$\boxed{L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx} \Rightarrow$$ Long. de curva de f de A a B.